Working with mathematics at the highest level is more of a vocation than a profession. Dennis Gaitsgory has been following his mathematical intuition for decades and has proved the geometric Langlands conjecture in collaboration with colleagues. This outstanding academic achievement has now earned him the Breakthrough Prize in Mathematics.

TEXT: DIRK EIDEMÜLLER

He puts down his coffee cup and welcomes me to the video call. Luckily, he's got enough time for our conversation. After all, Dennis Gaitsgory is a man in high demand. As Director at the Bonn-based Max Planck Institute for Mathematics, he is responsible for supervising his working group, but he has also recently entered the media spotlight – practically overnight. All the commotion that comes with this isn't necessarily to his liking.

Normally, specialists like him are only known to the initiated. Advanced mathematics is considered an extremely hermetic subject. Only a few colleagues worldwide specializing in the subject can understand what is published in certain mathematical disciplines. But last year, at the helm of a team of nine colleagues, he managed a great mathematical feat: finding the key proof for geometric Langlands conjecture – or at least a major part of this conjecture. For this achievement, he was awarded the Breakthrough Prize in Mathematics this year. The prize, which is worth USD 3 million, is given to a single recipient and was conferred during a ceremony in Santa Monica, California.

"I am, of course, delighted to have my work recognized," says the researcher, "but I should also stress that I would never have been able to come up with this proof were it not for the collaboration of my friends and colleagues – particularly Sam Raskin, who took on a large part of the work from 2022." Gaitsgory may be modest, but it should be said that he wasn't chosen as the prize winner entirely by chance. He devoted three decades of creative prowess to finding this proof, constantly pushing forward with the work, and inspiring and teaching his friends and colleagues – some of the co-authors were his students. Finally, he also took on a major role in the preparation of the final publications.

It all sounds a bit as if mathematics has always been his thing. But that's only partly true. "My father is a mathematician and would have been delighted if I'd been interested in mathematics as a youngster," says Gaitsgory. However, he preferred playing chess – and was very ambitious. "Chess was my favorite hobby from the age of ten to 14, and though I was good at math at school, I didn't have any real passion for it." When he was 14, he took part in the International Physics Olympiad, where he enjoyed puzzling over the difficult tasks. He planned to study physics.

But his path through life took a different turn. At that time, he was still living with his parents and younger sister in the former Soviet Union. While the maths and physics courses on offer there were

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VISIT TO

DENNIS GAITSGORY

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A great view, but with lots of hustle and bustle: Dennis Gaitsgory conducts research at the Max Planck Institute for Mathematics, which is opposite the Minster in the center of Bonn. To talk about mathematics in peace, he often looks to nature to get some seclusion.

very good, living conditions were not. When he was nearly 16, his family emigrated to Israel, moving to Tel Aviv. While a better life awaited Dennis Gaitsgory there, he was unable to speak Hebrew. But he was able to communicate in the language of mathematics from the very start. This made it easier for him to take part in a special course for particularly talented students, which the University of Tel Aviv was offering – albeit in mathematics, not physics. The program corresponded to that of a mathematics degree. "There were some math aces there who were a lot more advanced than me," remembers Gaitsgory. "It was there that I met Alexander Braverman, who was also from the Soviet Union. We became friends, and it was my contact with him and the course at the University of Tel Aviv that finally convinced me to switch to mathematics."

The youngsters enthusiastically immersed themselves in the subject and spurred one another on to greater and greater achievements. "It was a very inspiring environment, surrounded by the top talent in the region and taught by dedicated professors who really challenged us," says Gaitsgory. It was highly demanding. "Normal mathematics students had to get at least 60 out of 100 points in the final examinations, while students from our special course had to get 80, which was much more difficult!"

The university did this to ensure that only the best young people who were still of school age participated in this course. Around a dozen from each year group. It was also associated with the privilege of being able to miss many hours of normal lessons. Of course, they had to pass the final examinations – also in Hebrew. But considerable leeway was granted along the way. Ultimately, many of the students came from the Soviet Union and spoke hardly any Hebrew. "I have to say that, even in view of my experience in many other countries including Germany, I am still very impressed by the skillful educational practices of the school and university in Tel Aviv," says the researcher. "Many well-known scholars have emerged from the mathematics program in particular – and still continue to do so."

In his free time, he studied mathematics books or discussed mathematical theories and proof with his comrades-in-arms. He had already gained his high school leaving certificate by the age of 17. But Gaitsgory was approaching a particularly alarming time of life: military service, which is mandatory in Israel and lasts for three years for men. "I was now so deeply immersed in mathematics that I could scarcely imagine such a long time without it," he says. But his luck was in once again: there was no shortage of recruits at the time, due to the influx of immigrants, and he was able to avoid military service and devote himself to studying mathematics straight away.

"This transition meant I had no interruptions at all. I simply carried on from where I had left off at high school," he remarks. His supervisor at the time was Joseph Bernstein, an internationally renowned mathematician, whose students included some other present-day geniuses, alongside Gaitsgory. It was a small group of young people who dedicated themselves to mathematics in the manner of a sworn society. "We simply learned everything that Bernstein suggested and attended a fairly crazy number of courses," he says with a smile. "It didn't feel like a degree to us; we were just spending our entire time doing something that fascinated us: mathematics." At the start, it was all about studying basic mathematical concepts as broadly as possible.

During that time, he came into contact with the geometric Langlands conjecture, which was later to define his professional life. He knew very little about it when he heard a talk about its hypotheses. But he was immediately gripped by the subject. He sensed that this was a very deep and demanding area of mathematics that also aligned precisely with his personal interests and abilities. He specialized in these questions and devoted his doctorate to them.

But before he and I discuss the Langlands conjecture, we talk some more about his journey following his time in Tel Aviv. As is common in the postdoc phase, he moved between various places. He spent a total of two years in Princeton, where he studied the connections between abstract mathematical concepts and the theories of particle physics. This meant he had to engage with theoretical physics during his postdoc period. "That was extremely difficult for me, because the way in which mathematicians and physicists look at problems is very different," he says. Physicists simply calculate away and attempt to somehow understand events in nature. "We mathematicians, on the other hand, first consider whether a quantity is well defined and whether a statement is true or false," says Gaitsgory, adding half with regret and half with a laugh: "I've never managed to acquire the physics way of thinking!" He became a professor at the University of Chicago in 2001, and then

at Harvard University in 2005. It was there that he received an offer to become Director at the Max Planck Institute for Mathematics. After careful consideration, he decided in 2021 to accept.

The environment has lots to offer. In Bonn he enjoys cycling trips along the Rhine. When asked about the Institute, he answers somewhat evasively that the location is very central, right by the Münsterplatz. And the view, he says, is great. But it's obvious that he prefers peace and quiet, not the hustle and bustle and loud music of the city center. For that reason, he sometimes likes to meet visitors and colleagues in a natural setting where they can discuss mathematics in peace.

Furthermore, he is by no means only interested in his subject. "Of course, mathematics plays a very important role in my life, and passionate researchers like me can't just switch off from thinking about mathematics," he says. He adds that his wife and two sons have already gotten used to him being somewhat absent-minded in everyday life when he's thinking about mathematical problems. He says that this incessant brooding over scientific matters began in his youth and still continues today.

But mathematics is, of course, his greatest passion. However, explaining his research work isn't easy. A total of nine authors have now collaborated on the proof for the geometric Langlands conjecture. "Outside this team, there were probably only a good handful of experts worldwide who understood the proof," he estimates. However, the Breakthrough Prize is one factor that has piqued the curiosity of many colleagues, and Gaitsgory and his comrades-in-arms are now holding workshops globally to make the field more accessible. Other areas of mathematics might also benefit from the newly developed methods.

This proof has been compared with all manner of things in numerous media reports – including as the mathematical version of the Rosetta Stone. In its time, this enabled the deciphering of Egyptian hieroglyphics, since it also contained a translation into Greek and Demotic script. Elsewhere, the Langlands program, which was formulated as proof of the different variants of the Langlands conjecture, was termed the "great standardized theory" of mathematics. Gaitsgory is not satisfied with any of these terms, as they overstretch the content of his work.

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But this doesn't mean that there's nothing else in his life. For instance, in his free time he visits a climbing center. "I discovered this sport several years ago, and now go together with some of the students," he says. "We do talk about mathematical problems, of course, what else? But at the same time, we make sure that nobody falls," he adds with a smile. His second big hobby is the tango. He is a passionate dancer who can spend hours at dance events. There, he can switch off from mathematics and immerse himself entirely in the music and movement. He particularly loves the classic tango repertoire of the post-war era and sometimes acts as a tango DJ himself.

"Over half a century ago, Langlands came up with the idea that some very different areas of mathematics can be related to one another," he says. "But that doesn't mean that they can simply be translated into one another, like a dictionary translates one language into another."

The original Langlands conjecture related to the linking of arithmetic, in other words the theory of numbers, with analysis, which describes functions. As it emerged, such a conjecture can also be formulated for geometry. "Here, it's about investigating fundamental symmetrical relationships in defined, ab-

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stract geometric spaces," he explains. "It emerges that there are two different ways of solving these mathematical problems. Our proof now shows that these two ways are ultimately interchangeable." In other words, he says, it's not about a translation, but about proof of a deep-seated equivalence in abstract spaces. Only one aspect of the geometric Langlands conjecture is proved in the process, albeit an important one. It provides encouragement that advancement along this path is possible. Both the geometric and the original variant of the Langlands conjecture should benefit from this proof. Gaitsgory and his colleagues have opened up a whole host of doors for this branch of mathematics.

He has some unusual advice for budding female mathematicians in particular: "Just ignore the boys! Young men often have a very competitive nature, which is evident in their manner, and a ruthless mentality," he says. "You can't just prevent this, as it would inhibit their professional and academic development, but unfortunately it also makes young women feel ill at ease, particularly in abstract disciplines that are already highly selective." So he also wants teaching staff at schools and universities to make things easier for girls. For too long, he says, it has been difficult for women, who are less aggressive, to find a route into science.

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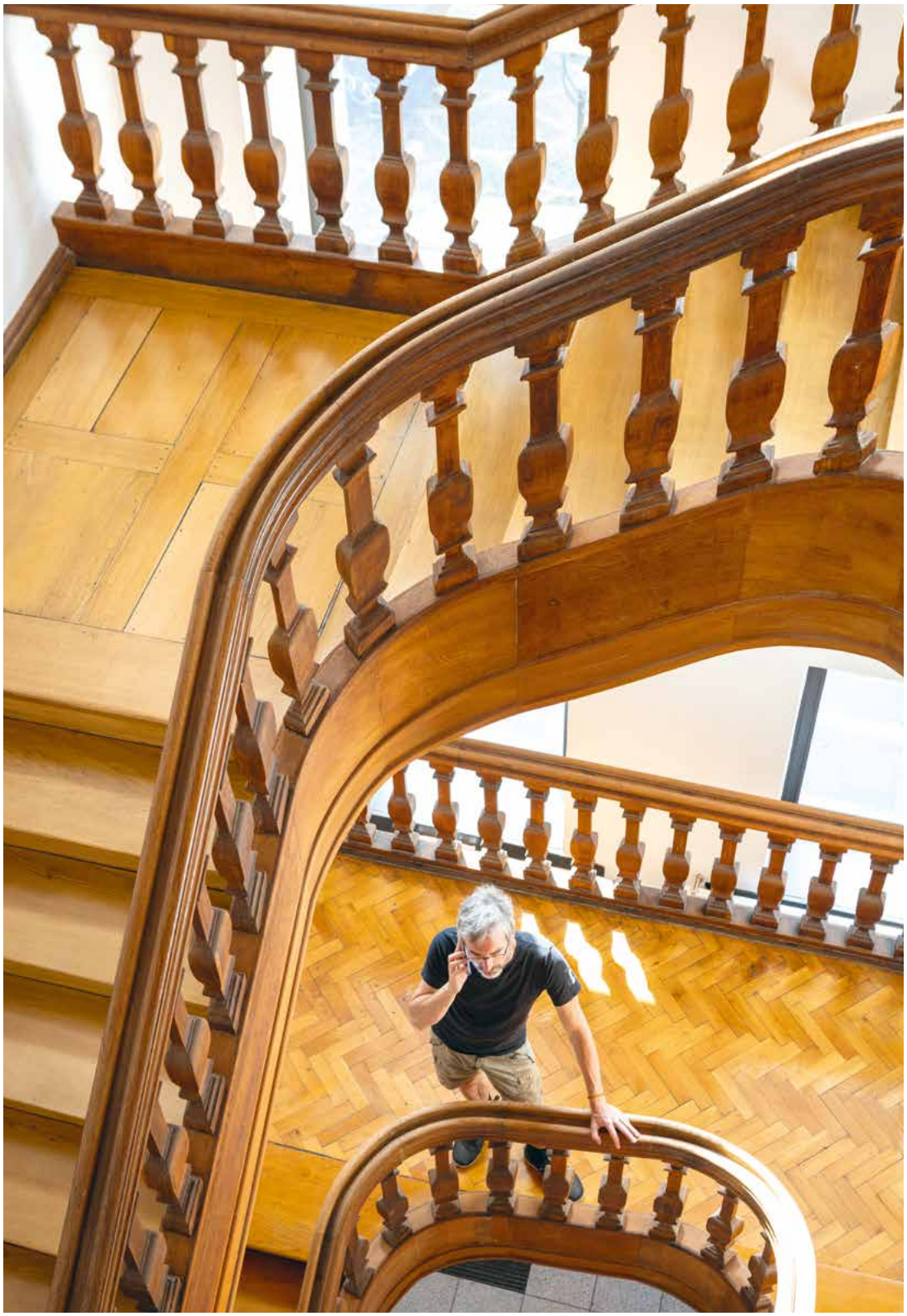
First of all though, the mathematical research community needs to digest the proof. The results are summarized in five publications stretching to almost 1,000 pages in total. "I wrote the majority of these myself, partly when I was laid up in bed after a skiing accident," he reveals. It took nearly one and a half years to get everything down on paper. The co-authors were also tasked with meticulously checking everything for coherence.

But what's next, now his life's work has been achieved? "I already have a host of ideas for new projects," he says. "Of course, I want to continue to work on the Langlands program, but not exclusively." One of his aims is to pass on his enthusiasm for mathematics and perseverance in research to the young generation. He advises budding mathematicians not to be intimidated by big problems. "Often, you make only tiny amounts of progress, step by step, for months at a time. That is the work of a mathematician," he reflects. And, he says, you don't need to worry if you don't have any insights into advanced mathematics at the age of 16. It doesn't matter if you're over 20 when this happens.

Above all though, academic work should be a pleasure. "A mathematical problem is like a giant puzzle, but there is no picture on the front of the jigsaw box that tells you what the final image should look like," he says. Instead, you have to develop the solution yourself. And you have to find satisfaction in this endless puzzle, as otherwise you won't be happy in mathematics in the long term. And so, he brings our conversation to an end in a jovial manner: "Finding the subject fun is both a necessary and a sufficient condition!" ←



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Lots to do: Gaitsgory finds satisfaction in mathematical puzzles, but as director he also has a lot of organizing to do. Since winning the Breakthrough Prize, his research has also gained a great deal of external attention.