



Many people switch their inner traffic light to red as soon as they hear the word mathematics: Stop, too difficult! Nevertheless, mathematics governs all of our thinking, and it is part and parcel of an astonishingly large number of everyday things and processes. **JURI MANIN**, Director Emeritus at the **MAX PLANCK INSTITUTE FOR MATHEMATICS** in Bonn, argues that mathematics is an important tool of human knowledge.

A short while ago, I was at a conference in Amsterdam and heard a manager give a talk in which he seriously put part of the blame for the crisis in the financial markets on us mathematicians. He leveled the accusation at us that our mathematical models were one of the reasons for this crisis. What he overlooked here was the fact that financial mathematics is based on statistical mathematics and thus predicts only probabilities and averages. In the end, the stock exchange is a game of chance. It is clear that many people, like this manager, are utterly naive in their use of these mathematical tools. They use them as a black box without understanding the mathematical models behind them. That explains why they are so surprised by sudden developments, such as those happening on the stock exchange, that may be mathematically improbable but not impossible.

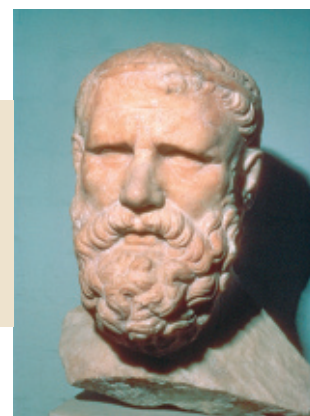
This example shows that it is worthwhile thinking about the many – often hidden – applications of mathematics in our daily life. This applies not only to the seemingly distant world of finance, which, nevertheless, has a quite direct influence on our life. Every technology is based on mathematically formulated theories and models, from computers and the Internet to aircraft and structural engineering. Doctors, social scientists, public opinion researchers, pollsters and governments use statistical mathematics methods when they test the effects of a new drug, draw up election forecasts or estimate future tax revenues. The natural sciences, of course, and first and foremost physics, make particularly intensive use of mathematical methods.

Many aspects of our world can be described precisely and intelligibly with mathematics. Since antiquity, this has led some mathematicians to believe that mathematical structures and forms are not simply thought up by humans, but exist independently of us. An indication of this could be the fact that new theories and models can emerge from completely abstract mathematical structures, and that these describe certain properties of

Mathematics is a tool of knowledge and, in turn, it provides our culture with powerful application tools. It produces abstract levers, so to speak, that we can use to move things in our real world with a high degree of efficiency. We are indebted to applied mathematics for progress and prosperity, but also for death and destruction in the form of new warfare tech-

Left: Design of a solar system: Johannes Kepler used geometric bodies to describe the orbits of the planets.

Right: A great mind, now unfortunately only in stone: Eratosthenes calculated the Earth's circumference fairly accurately using the lengths of shadows, approximations and ingenious geometry.



our world with precision. Physics knows many such examples: antimatter, black holes and particle spins were initially purely theoretical solutions to mathematical equations before it became clear that they really existed.

### THE DIALECTICS OF MATHEMATICS

So if mathematical structures were to exist independently of us, we mathematicians would not invent, but discover them. I cannot seriously defend the idea of the existence of such a platonic world of pure ideas outside of our minds. Emotionally, however, I strongly feel that we are not inventors, but discoverers. We can thus imagine our knowledge of the world as a hierarchy of models. Every model has two meanings: one that is located on the higher platonic level of purely mathematical language, and another one on a lower level, on which we can understand its relevance to the real world.

nology, the latter at least since the time of antiquity. Mathematics is thus also (mis-)used to simulate the effect of fragmentation bombs on human bodies, as testing on live pigs is prohibited for “humanitarian reasons.” Here, enlightenment and insight keep company with their most sinister counterpart.

Even in peaceful times, the naive use of mathematical tools as a black box holds certain dangers, not only on the stock exchange. Who, for example, thinks about what the Google search engine actually does when we use it to search for information on the Internet? This approach is comfortable, but it has its drawbacks. We do not realize how search engines filter information to deliberately manipulate us, and also to spy out our interests. A very similar thing happens with the protagonists – at least the more naive ones – on

the financial market. They look at their monitors and make investment decisions based on the information displayed, without having a very deep understanding of how it was generated. Often it turns out well. Occasionally, though, it goes disastrously wrong.

The financial world is based on numbers, and the history of mathematics starts with counting. Count-

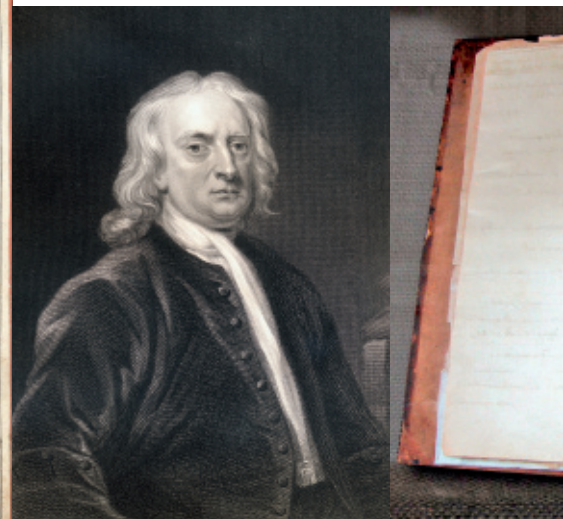
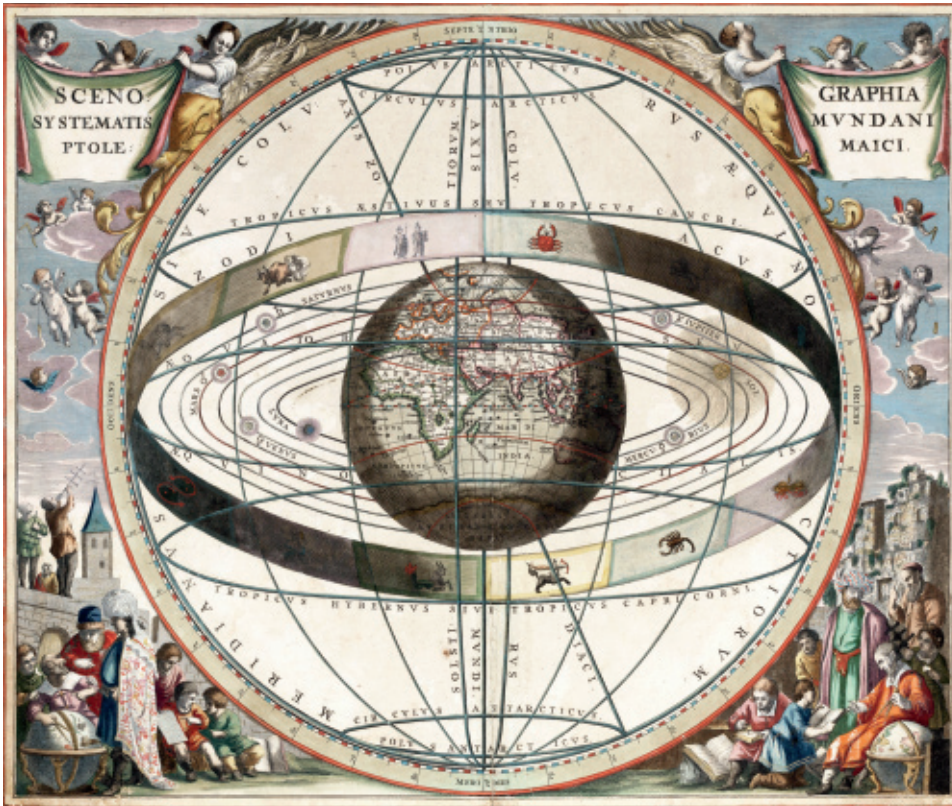
– numbers that we know as the simplest form of arithmetic. The numbers here still represented things or values from the material world: primeval mathematics was applied mathematics. It was only with the increasing degree of abstraction that pure mathematics evolved.

This and the invention of writing made it possible for complex administrative systems to develop. It was

utes). This now enabled the overseers to plan in advance how much time their gang of workers would need for a certain task. This work standard seems astonishingly modern, even today.

**DOOMED BY THE ROOT**

Ancient Greece was a turning point in Western culture for the development of mathematics. The word mathematics originates from the ancient Greek word μαθηματική τέχνη (mathēmatikḗ tékhnē), which in turn derives from μάθημα (máthēma) for



Ptolemaeus also described the orbits of planets mathematically – unfortunately, he got it wrong (left). Isaac Newton's law of gravity, presented in the Principia, is still valid, even if as a special case.

ing is a measurement, and the natural numbers 1, 2, 3 and so on provide the norm. Our forebears thus founded proto-mathematics, or primeval mathematics. The counting of cattle or containers filled with grain enabled humans to plan ahead for the first time. Since counting with fingers reached a limit at some stage, they used sticks, counting stones or notches. This made counting more abstract and powerful, because some symbols no longer represented only a 1, but 10 or perhaps 60. In addition, there were the basic arithmetical operations as simple calculating operations. This was the start of calculating with – initially only natural

the birth of more developed societies with a highly stratified division of labor. A good example is the administrative reform by the Sumerian king Shulgi, who presumably governed the country called Ur between 2047 and 1999 BC. Shulgi wanted to record the population's capacity for work and to use it more efficiently. The primary aim, of course, was to have a military fighting strength that was as large as possible, and that required an efficient economy to supply it. Shulgi introduced a standardized measure for the amount of work that could be accomplished by a group of workers in one sixtieth of the working day (about twelve min-

something learned, knowledge, science. The Greeks of antiquity also laid the cornerstone of pure mathematics. By dividing whole numbers, they arrived at rational numbers, and continued the game up to the question of the length of the diagonal in a square with a side length of 1. They found that it couldn't be a rational number! This was one of the great discoveries of the Pythagorean school.

Euclidian geometry, more than any other burgeoning branch of mathematics, represented pure mathematics at that time: suddenly there were areas extending into infinity, made of an infinite number of

infinitely small mathematical points. This was enormous progress in abstract thinking.

The resourceful Greeks readily used the new tools of pure mathematics for engineering and fundamental questions about our world. In other words, they turned the tables again. Eratosthenes of Cyrene (274 – 194 or 284 – 202 BC), for example, succeeded in estimating the circumference of the Earth with an accuracy that astonishes us today. The mathematician knew that in Syene in Egypt (now Aswan), the Sun did not cast a shadow at midday on the day of the sum-

mer solstice because, being so close to the equator, it was vertical at the zenith at noon. He compared this with the shadow cast in Alexandria on the same day, at the same time. Here, more to the north, the Sun was lower. With the aid of geometry, Eratosthenes concluded that the distance between Syene and Alexandria should correspond to one fiftieth of the circumference of the Earth.

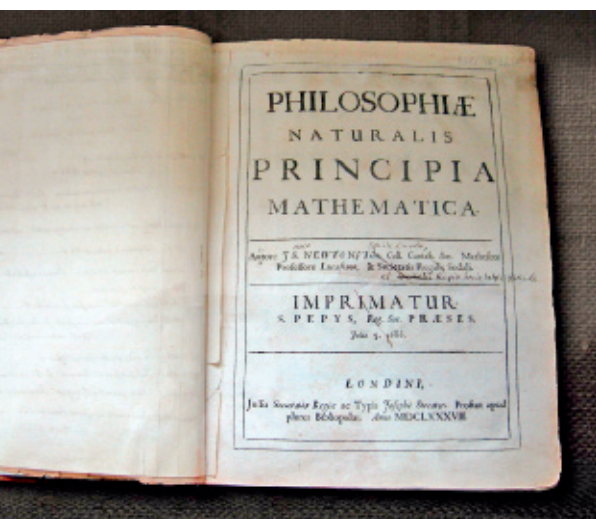
In order to be able to estimate the Earth's circumference from this angular measurement, the mathematician still required the distance between Alexandria and Syene. We now know that it is 850 kilometers. Eratosthenes put it at 5,000 Greek stadia. Perhaps he had this information from the highly developed administration of ancient Egypt, for example from average travel times of camel caravans or ships on the Nile. In any case, he thus arrived at a figure of 250,000 stadia for the circumference of the Earth. Converted to kilometers, this estimate is only one percent away from the modern value of the Earth's circumference.

In view of the imprecision of the data, Eratosthenes' estimate was amazingly precise – and of course he also had a little bit of luck. He used Euclidian geometry for his calculation, and made three assumptions: first, the Earth is a sphere; second, the Sun is very far away, so that its rays are incident in parallel; and third, Alexandria and Syene have the same longitude. Eratosthenes thus proceeded like a modern scientist: he developed a

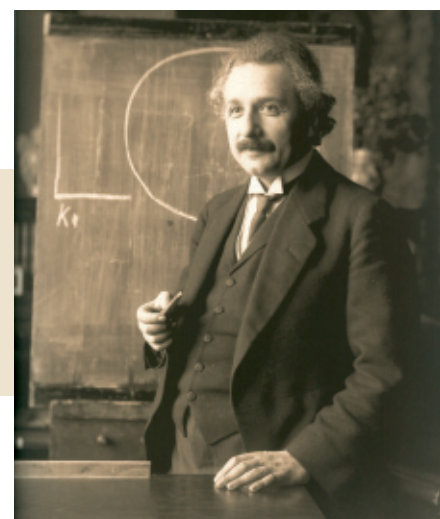
mathematical model, fed in measured data, and so obtained the Earth's circumference. This number was far beyond the scales used by people of that time – scales that were determined by the immediate environment in which those people lived. This finding about the Earth meant that mankind had made a huge intellectual leap.

between them. As a quantitative model, it can formulate processes in the micro-world into precise numbers. However, physicists must first correctly adjust more than two dozen "free parameters" – the screws of the model machinery, as it were. This tweaking of parameters is characteristic of models that describe a clearly defined phenomenon as accurately as possible.

Models are much more unassuming than theories. A good theory needs only a very small number of free parameters, or screws. In the best case, however, they should describe



**Lucky:** When Albert Einstein, seen here during a lecture in Vienna in 1921, formulated the general theory of relativity, the non-Euclidian geometry required was already in existence.



the entire world – or at least a highly idealized world. So theories arise from the concept that, beyond the material world, there is a reality that can be described mathematically. Physics again provides us some fine examples. Classical mechanics idealizes extended bodies by assuming that their entire mass is concentrated in small centers of gravity.

This assumption significantly simplifies the mathematical treatment of physical bodies. This allowed English physicist Isaac Newton (1643 – 1727) to formulate his famous law of gravity. This stated that two bodies with masses  $m_1$  and  $m_2$ , for instance the Earth and the moon, attract each other with the following gravitational force  $F = G \frac{m_1 \cdot m_2}{r^2}$ , where  $r$  is the distance between their centers of gravity, where their masses should

the practice of thinking in mathematical models spans from the epicycles, which Greek astronomer Claudius Ptolemaeus (approx. 100 – 175) used to describe the observed motion of the planets, to the modern Standard Model of elementary particle physics. The latter describes the elementary particles and the forces

### CREATING MODELS FOR PHYSICS

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PHOTOS - FROM LEFT TO RIGHT: OH / SIR ISAAC NEWTON (1643 - 1727) - PRIVATE COLLECTION - KEN WELSH - BRIDGEMANART.COM / F. SCHNUTZER

be concentrated. From a mathematical point of view, the constant  $G$  and the variables  $r$ ,  $m_1$  and  $m_2$  are free parameters, or the “screws” in the equation. However, the variable  $r^2$  contains an exact 2, not 2.000000003 or 1.99999995. Newton’s theory precisely specifies this 2.

### A VIEW OF LIFE FOR THE UNIVERSE

As soon as a new theory replaced Newton’s law of gravity, it wasn’t only the equation that changed, but the complete mathematical structure of the theory. That replacement was Albert Einstein’s general theory of relativity. It describes gravitation in much more general terms as a geometric result of the warping of four-dimensional space-time, which is

Bang is far beyond the human scale, it has become common knowledge that even children learn about. This is yet another huge intellectual leap in our understanding of the world.

The Big Bang is also a modern, mathematically formulated metaphor for the alpha, the beginning of our world. Cosmology also offers such a metaphor for the omega: According to the current state of our knowledge, our world will freeze up in a continuously expanding universe. So, in addition to models and theories, mathematical metaphors form a third kind of incredibly powerful tool of knowledge. A mathematical model is an invitation to develop models that can

complex of all organs, even though brain research still doesn’t understand how it functions.

Using the computer as a metaphor for the brain doesn’t do it justice at all. It is more suited to the functioning of individual nerve cells in the brain. After all, computers and neurons have an input and an output. The Internet is a much more precise and suitable mathematical metaphor for the brain as a whole: in the huge and open network, computers constantly make new connections; new participants come along, and others switch off – just like neurons in the brain.

Here, it could be argued that the computer and the Internet are not

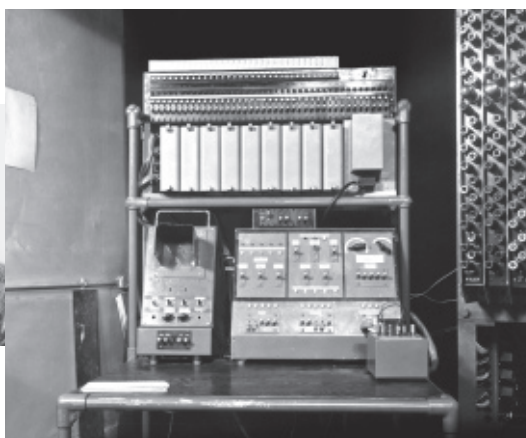
mathematical metaphors, but technical ones. But this is not the case: the computer, more than any other machine, is materialized mathematics. The mathematical foundation of modern – that is, freely programmable – computers is the so-called Turing machine. English mathematician Alan Turing (1912 – 1954)

devised an idealized image of a mechanical deterministic calculation that became famous as the Turing machine.

This machine could equally well imitate logical deductions. Before Turing, most metaphors related to mathematics and logic were linguistic ones: deductions and proofs were considered potential acts of (written) speech. Turing’s metaphor opened up our imagination and paved the way to the modern world of computers.

Finally, we can turn the tables in the best tradition of antiquity and ask ourselves what the Internet does with our brain. Does it change our scientific knowledge? What will happen if our thinking and knowledge become more and more complex, and are increasingly transferred to large databases and computer networks?

Pioneers of the computer age: Alan Turing taught computers – this is one of the first examples from 1951 – how to compute.



PHOTOS: SPL - AGENTUR FOCUS (2)

caused by the masses. The general theory of relativity doesn’t even have an equation for such a force: Newton’s law of gravity results only as a special borderline case.

So theories describe the world satisfactorily only until they are replaced by better theories. They are governed by our view of the world, which is itself in a state of flux. Newton’s view of the world still assumed that space and time, which were God-given, were absolute. Albert Einstein destroyed this view of the world with both of his theories of relativity at a time when general traditional views of the world were beginning to become shaky. The theories of relativity now offered a much more powerful mathematical toolbox. With this and with astronomical observations – that is, remote sensing – the cosmology of the 20th century has been able to expand our view of the world to the whole universe. Although the Big

be applied to reality (or, quite tangibly, to build machinery). A mathematical metaphor is an invitation to reflect on what we know.

### METAPHORS AS TOOLS

Metaphors compare something that we know quite well with something that we don’t know well. In this way, we make it more tangible for ourselves. A still-young mathematical metaphor is artificial intelligence. On the one hand, it comprises certain aspects of the technologically realized computer world. On the other, it is often used – together with computer hardware and software – as an image to illustrate how biological brains function. In this way, it gives us the feeling that we understand something about the most

I am convinced that the abstract thinking of mathematics will, in the future, continue to take place inside the heads of individuals. In any case, this is valid as long as the old science-fiction vision of chips that can be implanted into the brain does not become reality – a reality that would network us with other people and computers in a completely new way. But this is pure speculation.

There is no doubt that the Internet is changing the way we work, the

way we handle information and the way we communicate with each other. The knowledge available on the Internet is greatly increasing, news spreads much more quickly than before, and every individual is experiencing a permanent acceleration. But this development also harbors the risk of getting lost in distraction instead of reflecting at length. Paradoxically, it is precisely the Internet that could undermine creative thinking. The many mathematical black

boxes hidden in it would then lead to a loss of insight. However, since humans are adaptable, mathematics will surely remain a powerful knowledge tool in the future. ■

### HOW DOES GOOGLE WORK?

When search criteria are entered, Google produces, in simple terms, a list of Web pages. These pages contain the word or compound term sought. The search engine then has to sort this usually enormous list of pages according to the relevance of their content. But how does Google measure this relevance? The measure is provided by the hypertext links on the Web pages. One criterion for the importance of a Web page is the number of links to it from other Web pages. Google is still refining this selection strategy. It takes into consideration that not all links are of equal value: The link from an important Web page is given a higher weighting, while a link from an unimportant page gets a lower one. Each page thus transfers its own importance equally to all pages to which it links. The importance of the Web page being viewed results from the (weighted) links through which the other pages refer to it. Although this seems to lead to circular reasoning, it works. From a mathematical point of view, this strategy is well defined and is based on the Markov theorem.

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driving force behind modern civilization. He believes that as many people as possible should have access to this via popular science texts. He also enjoys illustrating articles.